Fibonacci, Pascal, and the number 5

STATEMENT : For any $n \in \mathbb{N}$, find the nth row of Pascal's Triangle, and mark off alternate elements, starting with the second.

E.g. for the case $n = 9$ we have the row :

1 <u>9</u> 36 <u>84</u> 126 <u>126</u> 84 <u>36</u> 9 <u>1</u>

with the required marked elements underlined.

Now multiply each marked element, in row order, by increasing powers of 5, starting at power 0, and add up all the answers.

So for $n = 9$ we have :

$$\begin{array}{rcccr} 9 & \times & 5^0 & = & 9 \\ 84 & \times & 5^1 & = & 420 \\ 126 & \times & 5^2 & = & 3150 \\ 36 & \times & 5^3 & = & 4500 \\ 1 & \times & 5^4 & = & 625 \\ \hline & & & & 8704 \end{array}$$

Now divide by 2^{n-1}.

In this example $2^{n-1} = 2^8 = 256$ and $\dfrac{8704}{256} = 34$.

And the answer is the nth Fibonacci number.

F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9
1	1	2	3	5	8	13	21	34

This works for any value of $n \in \mathbb{N}$.