VETTIUS VALENS AND LEWIS CARROLL

For the Maths-Jammers at Geraldine, 1st August 2021

"How many days are there in a year?" [asked Humpty Dumpty.]
"Three hundred and sixty-five," said Alice.
"And how many birthdays have you?"
"One."
"And if you take one from three hundred and sixty-five, what remains?"
"Three hundred and sixty-four, of course."
Humpty Dumpty looked doubtful. "I'd rather see that done on paper," he said.

Through the Looking-Glass, ch. vi

[This essay was begun as a quick synopsis of a short talk I gave at Geraldine on the date above. On the day I started writing, everyone in the country was ordered to stay at home to restrict any spread of the coronavirus disease ; writing became my project in isolation ; as the "lock-down" extended, so did the essay. It now includes a great deal of information not contained, or even hinted at, in the talk. I hope that the length may be forgiven, and that the content may prove not uninteresting to those who read it.]

1. INTRODUCTION

Vettius Valens was a writer in Greek who seems to have lived in the second century AD. Almost all that is known of him derives from his own work, which consists essentially of a series of books written over a period of twenty years or more, collectively called the *Anthologia*. He tells us that he lived in Antioch[1] (although clearly he travelled widely elsewhere) and that he occupied himself almost exclusively with the study of astrology. He seems to

[1] The ruins of Antioch lie close to the city of Antakya in modern-day Turkey.

have had a profound belief that everything occurring in the world is pre-determined and fated, and can be ascertained by an understanding of the movements of heavenly objects and of their positions at critical times (such as the moment of a person's birth). It was important to him that such movements and positions should be worked out as accurately as possible, using the best scientific knowledge then available.

Lewis Carroll[2] was a mathematics lecturer and Anglican deacon who lived in Oxford, England, in the nineteenth century, best known for two remarkable narratives, ostensibly written for children, entitled respectively *Alice's Adventures in Wonderland* and *Through the Looking-Glass and What Alice Found There*. These two men, far removed from each other in time, place and religion, have in common that they both proposed systems for determining, given some specific date in the past or the future, on what day of the week that date fell or would fall ; I call these systems *day-date algorithms*. This essay is to examine both systems and remark on their structure and soundness.

2. HOW A DAY-DATE ALGORITHM WORKS

The essential basis of any day-date algorithm is a known starting-point and a system for determining the modulo 7 equivalent of any offset from that point. For example, suppose we were interested in an algorithm for the present (21st) century only. We find by inspection that the last day of the

[2] This is a pseudonym adopted by Rev. C. L. Dodgson (1832-1898).

preceding century (31st December 2000) was a Sunday, and we call this the base-date. To find the weekday for any date thereafter, all we have to do is work out how many days have elapsed since then. Suppose we want to find out about 1st December 2019 ; we call the date about which we are inquiring the source-date. Then we compute that this day is exactly 6,909 days after the base-date. We divide this number by 7 and we find that it divides exactly. So an exact whole number of weeks have passed since the base-date, and therefore the source-date must have been a Sunday also. It is easy to see that if, when we divide by 7, there is a remainder of 1, then the day must be a Monday ; if a remainder of 2, then Tuesday ; and so on. It is very convenient to use a base-date falling on a Sunday, because the assignation of 1 to Monday, 2 to Tuesday, etc., works in happily with our usual idea of how the days of the week should be numbered.

Such a system is a start, but one obvious drawback is that the process of computing the number of days since the beginning of the century (6,909 in the present example) is likely to be tedious and productive of arithmetical error. And we are not interested in this total number as such, only in its modulo 7 equivalent. So our next step might be to note that 365, the number of days in an ordinary year, produces a remainder of 1 when divided by 7. Instead, therefore, of counting *all* the days that have elapsed since the base-date, it will suffice to count just 1 for *every whole ordinary year* that has passed.

We must remember to take account of leap-years. 366, the number of days in a leap-year, produces a remainder of 2 when divided by 7. Therefore, for every whole leap-year that has passed since the base-date, we must count 2. Probably the easiest way to put this into practice is to count 1 for every whole year without distinction, and then an additional 1 for every leap-year. Our chosen base-date, coming at the end of a leap-year, is again most convenient here : there will be 1 to add for every complete period of 4 years since the century began.

Returning to our example, then, we have that on the source-date (1st December 2019) there have been 18 full years since the base-date. We compute the number of leap-years by dividing this number by 4 : we take the quotient and discard the remainder. So for the 18 years we count 18 + 4 = 22. We now determine that 1st December is the 335th day of the year. We compute 22 + 335 = 357 and we see again that 7 divides this exactly, so we know that our date was a Sunday.

As we shall see, this system can still be improved upon. The calculation of 1st December as being the 335th day of the year is still a little awkward. But with this basic knowledge of how a day-date algorithm works it is now possible to consider the system of Vettius Valens.

3. THE SYSTEM OF VETTIUS VALENS

Here is the system as it is presented in Valens's own work[3] :

> A handy method for the seven zone system. For the week proceed as follows : take the full years of the Augustan era and the leap years, and add to that sum the days from 1 Thoth to the birth-date. Then subtract as many sevens as possible. Count the result off from the sun's day, and the birth-date will belong to the star at which the count stops.
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> The order of the stars with respect to the days is : Sun, Moon, Mars, Mercury, Jupiter, Venus, Saturn. The arrangement of their spheres is : Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon. It is from this latter arrangement that the hours are named, and from the hours, the day of the next star in sequence.
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> For example : 4th year of Hadrian, 13 Mechir (in the Alexandrian calendar), the first hour of the night. The full years of the Augustan era are 148, the leap years are 36, and from 1 Thoth to 13 Mechir are 163 days. The total is 347. I divide by 7 for a result of 49, remainder 4. Starting from the sun's day, the count comes to Mercury's day. The first hour of that day belongs to Mercury.

The first point to understand about this is that Valens worked at all times with the Alexandrian calendar, in the form that it took in the days of Cæsar Augustus. This does not in fact make things as difficult as they might seem, because the modification that took place under Augustus had the effect of anchoring the calendar of Alexandria to that of Rome, so that in every year the same pairs of dates became equivalent to each other. This calendar of

[3] *Anthologia* 1.10 ; the translation of Professor M. T. Riley.

Rome is usually called the Julian calendar,[4] and was the direct ancestor of the Gregorian calendar[5] in general use today. See Appendix for details of the distinction between the two calendars.

The Alexandrian calendar had 12 months, all of which comprised 30 days each ; the first month was called Thoth, and the last month was called Mesore. After Mesore there were then a further 5 days which were not considered to belong to any month ; these days were called *epagomenal* (meaning "additional"). This made up a whole year of 365 days. The modification imposed by Augustus was the introduction of leap-years : every fourth year there would be 6 epagomenal days rather than 5.

In ordinary years the first day of the year, 1 Thoth, was equivalent to the Julian 29th August[6] ; the epagomenal days were the days 24th-28th August. When there was due to be a leap-year in the Julian calendar, the extra epagomenal day in Egypt was intercalated in the August *before* the Julian leap-year. This caused 1 Thoth to become equivalent to 30th August in those years, and there would be a temporary dislocation of one day in the Egyptian-Julian date equivalences lasting until the following February, when the extra day was intercalated at Rome and the balance was restored.

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[4] Named after the dictator C. Julius Cæsar who introduced it.

[5] Named after Pope Gregory XIII who introduced it.

[6] I give Roman dates in modern notation. The Romans themselves used a complicated system of denoting dates by reference to the Kalends (the first day of each month), the Nones (the fifth or seventh day) and the Ides (the thirteenth or fifteenth day). Dates which were not one of these "dividing days" were counted backwards and inclusively from the next such day in the future. For example 29th August would have been denoted *iv Kal. Sept.* (4 days before the Kalends of September). This Roman notation is extremely interesting, but further discussion is unfortunately beyond the scope of this essay.

While Valens himself may well not have thought in the same way as we do about a base-date for his system (he certainly makes no mention of one) it is still fairly clear that his computations ran along the lines I have described in the last section : that is a count of 1 for every “full” year up to the source-date, another 1 for every leap-year, then add the number of days from the start of the year to the source-date, and finally divide by 7. There are, however, still some questions to answer.

First of all, we need to decide what Valens means by the “Augustan era”. Of course we can see immediately the problem with which he must have been faced : there did not exist in his time any standard system for the numbering of years. Writers occasionally refer to years *ab urbe condita* (years since the supposed founding of Rome by Romulus and Remus), and to numbered Olympiads (periods of 4 years during the first of which was celebrated the great Olympic festival with games) ; but such references are the exception rather than the rule.[7] It was much more common to designate a year by reference to the Roman consuls who held office during it, or by the year of the reign of some ruler. Although this sort of usage does not work well with a general dating algorithm, Valens seems to have found that he had little choice but to use the numbers of years of the reigns of the various Roman Emperors since the base-date.

[7] The historians Eutropius and Orosius, for example, writing in the third or fourth centuries, use a system of dates AUC ; Eusebius of Cæsarea uses Olympiads. Such usages are hardly ever found in official records, however.

In the section I have quoted Valens provides no explanation of "Augustan era". However, in a later section,[8] which deals with a system for determining the phase of the moon from any given date, he uses the term again ; and this time there appears in his text a list of Roman Emperors together with the whole number of years to be attributed to the reigns of each.[9] Here is the table for the first twelve Emperors :

Emperor	Years of reign	Date of death	Running total of years
Cæsar Augustus	43	19th August 14	43
Tiberius	22	March 37	65
Gaius (Caligula)	4	January 41	69
Claudius	14	October 54	83
Nero	14	June 68	97
Vespasian	10	June 79	107
Titus	3	September 81	110
Domitian	15	September 96	125
Nerva	1	January 98	126
Trajan	19	8th August 117	145
Hadrian	21	July 138	166
Antoninus Pius	23	March 161	189

[8] *Anthologia* 1.19.

[9] Serious questions have been asked about the authenticity of this table. Except in a single sentence immediately introducing it, the text does not mention it, and no information is given about the historical sources on which it is based. The last name in it is that of Philip, Emperor AD 244-249 ; as Philip's reign was about 80 years after Valens was writing it is certain that at least part of the table was interpolated by later copyists. The table has five columns, of which the third, fourth and fifth (all omitted here) are concerned with the moon phase computation : these columns present major arithmetical problems throughout. It is easy to suspect that the entire table is an interpolation. For a recent discussion see Declercq G. "The Royal Canon in the *Anthologies* of Vettius Valens" in *Zeitschrift für Papyrologie und Epigraphik* 204 (2017) pp. 221-228. Having said all this, the table does seem to work well with the day-date algorithm.

The first two columns (in black) are as they are given in the text. I have supplied the third and fourth columns (in blue) for ease of computation and commentary. The years in the third column are all AD.

The table suggests that what Valens means by the "Augustan era" is the time commencing at a point 43 years before the end of Augustus's reign. This fits in well with our historical knowledge. Augustus took military control of Egypt during the months June to August in the year we would call 30 BC, following the deaths of Marcus Antonius and Cleopatra, so the year starting on 29th August immediately thereafter was the first year of the modified Alexandrian calendar with leap-years. It makes good sense, from an Egyptian point of view, to suppose that this date is to be regarded as the start of the "Augustan era".

A more difficult problem is the basis on which the table attributes years in which the imperial crown changed hands. I conjecture that a year is to be counted to an Emperor if and only if *he was reigning at the end of it*. Part-years at the beginning of his reign count to him on this theory ; part-years at the end do not. This would explain very well, for example, why Titus (June 79 to September 81) has three years attributed to him ; and in fact it works for all but two of the Emperors in the table. Augustus died ten days before the new year on 29th August 14, and Nero about two months before the new year on 29th August 68 ; in both of these cases the part-year has been attributed to the outgoing Emperor (and in both cases deducted from the correct total for the new Emperor, thus keeping overall totals in balance). It is, of course, perfectly possible that the author of the table was working with different information as the exact dates of these Emperors' deaths, which were no longer within living memory when he was writing.

4. VALENS'S WORKED EXAMPLE

At this point, it is easiest to turn to the practical example computation which Valens provides. This is 13 Mechir (the sixth month of the Alexandrian calendar) in the fourth year of Hadrian ; we would call it 8th February AD 120.[10] In ordinary years 13 Mechir would be equivalent to 7th February, but this particular date occurs during the period of 1-day dislocation between the Alexandrian and Julian calendars, occasioned by the leap-year, as explained in the preceding section.

Valens tells us that in the fourth year of Hadrian there were completed 148 full years of the Augustan era. It seems fairly clear that he has found this number by summing up the years of all the Emperors before Hadrian (that is Augustus to Trajan inclusive), which is 145 (the fourth column of running totals in the table has been supplied to avoid unnecessary extra arithmetic in this part of the calculation), and then adding the 3 years of Hadrian that have already been completed. This makes 148 as required.

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[10] Valens uses this date 13 Mechir 4 Hadrian in a number of other examples throughout the course of his work. It is not unreasonable to conjecture that it was in fact his own date of birth.

Valens now adds the number of leap-years, and this is rather more problematic. He gives the number as 36, but does not explain how he ascertains it. It is not what we expect, because dividing 148 by 4 produces a quotient of 37. I hazard a conjecture on this, as follows.

There is some evidence to suggest that the sequence of leap-years (that is, one every fourth year) was not followed properly in the first 50 years or so of the Julian calendar. Julius Cæsar, as *pontifex maximus*, the official in the Roman priestly college having responsibility for calendric matters, introduced the new calendar with effect from 1st January in the year we would call 45 BC, but he was assassinated the following year without having had a good opportunity to see that his innovations were implemented properly. The new *pontifex maximus* was M. Æmilius Lepidus, who does not traditionally enjoy a good reputation for competence : the Shakespearian image of this "slight, unmeritable man"[11] dies hard. In any event he was exiled from Rome in 36 BC, but not deprived of his office, a circumstance which must have created an authority vacuum, since he proceeded to live on for another quarter of a century in peaceful obscurity.

Solinus and Macrobius[12] are the writers who give specific detail about the calendar problem during Lepidus's pontificate. They say that additional days were intercalated every *third* year, instead of every fourth ; that this continued for a period of 36 years ; and that, to correct the error, Augustus (who became *pontifex maximus* around 9 BC) then ordered twelve years to

[11] *Julius Cæsar*, iv 1.
[12] Solinus, *De Mirabilibus Mundi* 1.45-47 ; Macrobius, *Saturnalia* 1.14.13-15.

pass without any intercalation at all. There is some divergence in scholarly opinion as to exactly which years were leap-years, but a fairly standard reconstruction is put forward by Robert Hannah, who explores the matter in some depth.[13] This places leap-years in every third year from 42 to 9 BC inclusive (thus, as it chances, always in years divisible by 3), with intercalations being resumed, now every fourth year, with effect from AD 8.

On this basis it can be seen that there will always be one fewer leap-year in the "Augustan era" than would be expected. Take the situation as at 1 Thoth AD 11. There will have been completed exactly 40 years of the Augustan era. We would expect, under normal circumstances, that 10 of these would have been leap-years. But (on Hannah's reconstruction) the only years to have been leap-years were 28, 25, 22, 19, 16, 13 and 10 BC (remember that the intercalation in Alexandria is made during the year *preceding* the intercalation at Rome), and AD 7 and 11 (the day immediately before 1 Thoth AD 11 will have been intercalated). This makes just 9 leap-years altogether.[14]

Returning to Valens's example, the rest of his computation is straight-forward. Note that it is much easier for him to calculate how many days have passed since the beginning of the year than it would be for us, because every Egyptian month has 30 days. When he is dealing with the thirteenth day of the sixth month it is the work of a moment for him to multiply 5 (months already completed) by 30 and then add 13. We would probably take

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[13] *Greek and Roman Calendars* (2005) London, pp. 118-122.

[14] In his argument Hannah does not in fact mention Valens at all, although this analysis of Valens's algorithm very much supports his reconstruction.

rather longer to compute that the number of days from 30th August to 8th February (inclusive) is 163.

Valens now sums up 148 + 36 + 163 = 347, and notes that this number produces a remainder of 4 when divided by 7, which for him indicates Mercury, or Wednesday.[15] His system clearly comprehends that a remainder of 1 assigns the result to Sunday, 2 to Monday, and so on. This is different from the system I described above, but it makes entire sense in the context of the classical world, where Sunday was almost universally accounted as the first day of the week.[16] If Valens did think by reference to a base-date, he would have seen how it follows from all this that the day immediately preceding the start of his Augustan era, 5 Epag. (= 28th August) in the year we would call 30 BC, was a Saturday.

As we will see, if we project the Julian calendar back to the reign of Hadrian using modern notation, we find that 8th February AD 120 was indeed a Wednesday. Sacha Stern has noted[17] that this correspondence gives us good reason to think that, over all the calendric and political vicissitudes of the

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[15] Valens denotes the days by reference to the "stars" Sun, Moon, Mars, Mercury, Jupiter, Venus and Saturn, in that order. The first, second and seventh of these clearly equate to our Sunday, Monday and Saturday, respectively. In typical compromise fashion, the English language has replaced the other four planetary names with the names of Anglo-Saxon deities which have a rough equivalence : Tiw, Woden, Thunor and Frige. But the names in French (for example) of these four days clearly shew the original relationship : *mardi*, *mercredi*, *jeudi* and *vendredi*.

[16] Cf., for example, Jn. xx 1 (an account of the first Easter Sunday morning) : "The first day of the week cometh Mary Magdalene early."

[17] *Calendar and Community* (2001) New York, p. 107.

world in the last 1,900 years, the cycle of the seven-day week has remained entire and undisturbed throughout.

5. USING THE ALGORITHM OF VALENS TODAY

Day-date algorithms do not expire, although provision has to be made for calendar changes. It is perfectly possible to find the week-day for a modern date using Valens's algorithm, thus shewing the system's continuing mathematical validity. We will use our existing example of 1st December 2019.

We can find the year of the Augustan era (which must always be computed as of the year commencing on 29th or 30th August preceding the source-date) by adding 29 to the year AD. We can find the number of leap-years as we did before, by adding 1 to the year AD, dividing by 4 and discarding the remainder ; but to these we must add 6, which represent the 7 leap-years that took place in the Augustan BC years (see section 4) minus 1 for the leap-year which was suppressed in AD 4. It is also necessary to find the Alexandrian calendar equivalent of our source-date. As it is a Gregorian calendar date we must first shift it back 13 days to allow for the present disjunction between Gregorian and Julian calendars (see Appendix) ; then we convert to Alexandrian reckoning using the year commencing in the preceding August.

So for the years we have 2019 + 29 = 2048. For the leap-years we have 2020 divided by 4 produces a quotient of 505 (no remainder to discard on this occasion). As explained we then compute 505 + 6 = 511. Shifting 1st December back by 13 days we have its Julian equivalent 18th November. As the next year (2020) is a leap-year, the Alexandrian year began on 30th August 2019. So the Alexandrian date equivalent to 18th November is 21 Hathyr (the third month) and it is easy to compute that this is day 81 of the Alexandrian year.

In total we have 2048 + 511 + 81 = 2640. Divide by 7 and the remainder is 1, which represents Sunday in Valens's system, as expected.

6. THE SYSTEM OF LEWIS CARROLL

Lewis Carroll's algorithm was published on 31st March 1887 in *Nature* magazine. Here is what he has to say :

> Having hit upon the following method of mentally computing the day of the week for any given date, I send it to you in the hope that it may interest some of your readers. I am not a rapid computer myself, and as I find my average time for doing any such question is about 20 seconds, I have little doubt that a rapid computer would not need 15.
>
> Take the given date in 4 portions, viz. the number of centuries, the number of years over, the month, and the day of the month.
>
> Compute the following 4 items, adding each, when found, to the total of the previous items. When an item or total exceeds 7, divide by 7 and keep the remainder only.

> *The Century-Item* : For Old Style (which ended September 2, 1752) subtract from 18. For New Style (which began September 14 [1752]) divide by 4, take overplus from 3, multiply remainder by 2.
>
> *The Year-Item* : Add together the number of dozens, the overplus, and the number of 4s in the overplus.
>
> *The Month-Item* : If it begins or ends with a vowel, subtract the number, denoting its place in the year, from 10. This, plus its number of days, gives the item for the following month. The item for January is '0' ; for February or March (the 3rd month) '3' ; for December (the 12th month) '12'.
>
> *The Day-Item* is the day of the month.
>
> The total, thus reached, must be corrected by deducting '1' (first adding 7 if the total be '0') if the date be January or February in a Leap Year : remembering that every year, divisible by 4, is a Leap Year, excepting only the century-years, in New Style, when the number of centuries is *not* so divisible (*e.g*. 1800).
>
> The final result gives the day of the week, '0' meaning Sunday, '1' Monday, and so on.

Considering that these words were written well over a century ago, they are very straight-forward. It is interesting to note the subtle grammatical distinction intended when a number expressed digitally is enclosed in quotation marks, for example '12'. We must probably forgive Carroll for some slight imprecision with his terminology. He seems to use the words *overplus* and *remainder* interchangeably for what is left over after a division sum, while the latter word is given an alternative meaning as the result of a subtraction sum.

But leaving aside these minor considerations, we can probably see already the fundamental similarity of Carroll's system with those we have already considered. In fact, Carroll makes no mention here (or, as far as I know, anywhere else) of Vettius Valens,[18] although we may note with some resignation that he has followed Valens faithfully in omitting to provide any explanation of how his system works. I will try to rectify that here.

Taking his "items" in reverse order, we see that Carroll has separated out the "day of the year" portion of the calculation into two separate components, one for the month in which the source-date falls, and the other for the day of the month. For the month itself, Carroll *effectively* uses the following table.[19]

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[18] It is possible to speculate that Carroll felt some scruples about introducing his readers to the pagan astrologer Vettius Valens. Carroll took his position as an Anglican deacon very seriously, and never allows himself to say a word that might possibly be construed as a slight on the Church or its doctrine. Consider, for example, the "Easter letter" which he inserted into the great nonsense-poem *The Hunting of the Snark* ; the complete absence of bishops from the chess-game at the heart of the plot in *Through the Looking-Glass* ; and the fact that, when he discovered that the talking passion-flower with which Alice converses in the same book was so called with reference to the Passion of Christ, he quickly transformed it into a tiger-lily. On this last point, see Martin Gardner, *The Annotated Alice* (definitive edition) (2000), New York, p. 157.

[19] Carroll says explicitly that his method is intended for *mental* computation ; that is, he wants to have a system which is *portable*. He wishes to avoid having to depend on the availability of some reference document, like a table. He therefore memorises the counts for January, February, March and December (using 12 for December, which is equivalent to 5 and is easier to remember, December being the 12th month). For the rest he proposes this elaborate process involving the months beginning or ending with a vowel. What he says is arithmetically correct, but may seem somewhat cumbersome to us today.

Month	Count	Month	Count
January	0	July	6
February	3	August	2
March	3	September	5
April	6	October	0
May	1	November	3
June	4	December	5

It makes sense that January will have count 0, because the number of the day in January is the same as the number of the day in the year. It can be seen that each subsequent count is obtained by adding the count of the preceding month to the number of days in that month and reducing the result to modulo 7. After that, of course, the count for the day is simply the day of the month itself. For example 1st August has total count 1 + 2 = 3, 1 for the 1st day of the month, and 2 from the table for August.

For his "year-item", Carroll introduces a twist on the basic system I described in section 2 above. I said that one possible way is to divide the number of the year by 4, discarding the remainder, and then add dividend and quotient together. Thus (for example) Year 21 produces a result of 21 + 5 = 26. Carroll prefers to divide by 12 first, and then divide the remainder by 4, discarding only the *second* remainder. He then sums up the first quotient, the first remainder, and the second quotient. On this system we have the computation (for Year 21) 1 + 9 + 2 = 12. Clearly 12 and 26 are equivalents in modulo 7, so all is well. We can find, on analysis, that this always works because 12 years comprise exactly 4,383 days (12 times 365, plus 3 intercalated days) : this number is equivalent to 1 in modulo 7, so it suffices to count just 1 for every whole period of 12 years before dividing by 4. This aspect of Carroll's system is debatably easier to manage as the numbers of the years get larger.

An important point to note about the year-item is that Carroll uses the number of the year in which the source-date falls, as opposed to the number of full years completed at the beginning of that year. By doing this, he is adding in an extra 1 when the source-date is in a leap-year, even if is a date before the actual intercalation. He provides for this by directing the subtraction of 1 from the final total, whenever the source-date is in January or February (including the intercalated day itself) of a leap-year. There is also a consequence for the base-date and the century-item, to which we will now turn. We must consider the Old Style and the New separately.

7. The New Style

By the New Style, Carroll means the Gregorian calendar[20] ; this was a standard designation for it in nineteenth century Britain, and is still occasionally used today. Now, as we know, the first day of the present

[20] The term "Gregorian calendar" did not come into formal usage in the English language until relatively recently. When the new calendar was (finally) introduced into Britain in 1752, there was still a great deal of popular feeling in the country hostile to the Roman Catholic church, so it would have been politically inexpedient to give anything an official name derived from a Catholic Pope. The statute introducing the change names the old calendar specifically as the Julian calendar, but its replacement is simply called the "new calendar". When this statute was later given a formal "Short Title", which occurred in 1896, less than ten years after Carroll wrote his algorithm, it was called the Calendar (New Style) Act 1750.

century, 1st January 2001, was a Monday. We also know that 1st January, in a year ending "01", is going to produce, in Carroll's algorithm, a combined total for year, month and day-items of 2 (1 for the year, plus 0 for the month, plus 1 for the day). To make the system work, therefore, the century-item (for this century) must be 6 ; this will produce a final total of 8, which leads to a result of Monday.

This century will contain 36,524 days (100 times 365, plus 24 intercalated days : the intercalation will be suppressed in 2100, the last year of the century). This number is equivalent to 5 in modulo 7, so we see that the century-item for *next* century (the 22nd) will be 4. By the same argument the item for the 23rd century will be 2, and for the 24th century, 0. At the end of the 24th century, in the year 2400, the intercalation will *not* be suppressed, so the century will have an extra day, bringing it to a total equivalent to 6 in modulo 7 : this will bring the century-item back to 6 for the 25th century. Note from all this how the *quadricentennium*, the period of 400 years on which the Gregorian calendar is based, has always a number of days (146,097 in fact) which is an exact multiple of 7 ; therefore the cycle of weekdays in any given century will always be exactly repeated four centuries later. It seems that this convenience only arose as a matter of chance.

These results could be summed up in the following table :

Century within Quadricentennium	Count
First	6
Second	4
Third	2
Fourth	0

It can easily be seen that the calculation given by Carroll (divide by 4, subtract remainder from 3, and double) produces the results shown the table. It is important to remember that, when talking of the "century" of a year, Carroll means its number obtained by removing its last two digits. Thus, although the present century is the twenty-first, its number to be used in this algorithm is 20.

Checking all this with our example (1st December 2019), we have century-item 6, year 1 + 7 + 1 = 9, month 5, and day 1 ; total 21 ≡ 0 (mod. 7) ; result Sunday as expected.

8. The Old Style

In the Julian calendar, all centuries include 25 intercalated days, and therefore 36,525 days in total, equivalent to 6 in modulo 7. Therefore Carroll's century-item must diminish by 1 every century. In this system it will take 700 years, rather than 400, to cycle through all the possible counts. Again we construct the appropriate table, as follows :

Century within Septicentennium	Count
First	6
Second	5
Third	4
Fourth	3
Fifth	2
Sixth	1
Seventh	0

The back-projected (or *proleptic*) Julian and Gregorian calendars were in synchrony in the third century (see Appendix), when the Gregorian count was 2. This century must therefore be accounted the fifth in the Julian septicentennium (so as to produce the same count) as must the tenth and seventeenth centuries. It is now easy to see how Carroll's direction to subtract the century's number (again obtained by removing the year's last two digits) from 18 will always work. Note that, if we want to find the day for a Julian date later than the nineteenth century, we can easily do this by subtracting from 25 (or some higher modulo 7 equivalent) instead.

Let us finally check this with the example of Vettius Valens, which was 8th February AD 120. Century-item 18 – 1 = 17 ; year 1 + 8 + 2 = 11 ; month 3 (as per table in section 6) ; day 8 ; finally remember to subtract 1 for a date in January or February of leap-year. Total 17 + 11 + 3 + 8 – 1 = 38 ≡ 3 (mod. 7), which represents Wednesday, as expected.

9. Conclusion

Day-date algorithms are not mathematically difficult, but they can have a cumbersome feel which makes them seem less simple than they are. This is probably inevitable given the irregularity of the lengths of the months, the lack of fit between the lengths of the week, month and year, and the incidence of intercalation. There have to be a quite a lot of rules, to cater for different situations. However, the algorithms of Vettius Valens and Lewis Carroll are both structurally sound ; they can still be used today, and, subject to proper future administration of the calendar, should continue to be serviceable into the indefinite future. In the electronic age, one wonders if general standards of mental arithmetic are quite what they were in Carroll's lifetime ; nevertheless his suggestion that a good "calculator" could work the algorithm in 15 seconds or less does not seem unreasonable.

APPENDIX

The Distinction Between the Julian and Gregorian Calendars

In both the Julian and Gregorian calendars the names and lengths of the months are the same ; the names are mostly derived from those in the former pre-Julian or Republican calendar of Rome. Both also have leap-years in which an extra day is added (or *intercalated*) into February[21] ; this happens

[21] It is interesting to note that the practice of naming the intercalated day 29th February is a relatively modern phenomenon. Prior to about the middle of the seventeenth century it was usual in most places to observe the intercalation by repeating the date 24th February.

whenever the AD number of the year is an exact multiple of 4. The only difference between the calendars rarely makes itself apparent, although it leads to an increasing cumulative disjunction.

In the Julian calendar every fourth year, without exception, is a leap-year and contains 366 days. In the Gregorian calendar, in every period of 400 years, intercalations are omitted (or *suppressed*) three times, in the 100th, 200th and 300th years (*not* the 400th year) of the period respectively. This suppression last happened in the year 1900 ; it will not happen again until 2100.

The Gregorian calendar was first proposed in the 1580s and is now the predominant calendar of the world. Many places were, however, initially reluctant to take it up. In Britain, for example, the shift was made on the day after Wednesday 2nd September 1752. The disjunction between calendars then amounting to 11 days (see next paragraph), the days 3rd to 13th September inclusive were all suppressed in this year, and the New Style commenced on Thursday 14th September. This explains Carroll's reference to these dates in his system. Note that there was never any place or time wherein the change of calendar was allowed to disturb the cycle of the seven-day week.

The New Style reduces the mean length of the calendar year from 365.25 to 365.2425 days. The latter number more accurately reflects the length of the

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This follows actual practice in ancient Rome, where the intercalated day was denoted *bis vi Kal. Mar.* (6 days before the Kalends of March, a second time). As to Roman date notation see n. 6. This is the origin of the word *bissextile*, an old-fashioned word in the English language for *leap-year*, and still the current word in French.

solar tropic year.[22] The reform bases itself on the idea that, if projected back in time, the two calendars would have been in synchrony during the third century. Since that time the Gregorian calendar has suppressed intercalations in each of the years AD 300, 500, 600, 700, 900, 1000, 1100, 1300, 1400, 1500 (these ten occasions retrospectively, so to speak), 1700, 1800 and 1900. At the present time, Gregorian and Julian dates are therefore 13 days apart. A notable demonstration of this occurs when many Orthodox Christians, still using the Julian calendar, celebrate Christmas on 7th January, 13 days after the Gregorian 25th December.

CHRISTOPHER JOHN RICHARD HEXT

September 2021

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[22] That is, the seasonal year, typically measured from one moment of the (Northern) Spring equinox (when the Sun, in its passage round the ecliptic, crosses the celestial equator going northwards) to the next, which is in fact about 365.24219 days (accuracy greater than this is not meaningful). The Gregorian mean calendar year is therefore still about 27 seconds too long. This error will accumulate to a whole day after about 3,200 years.