Poker Rankings

Consider forming a 5 card poker hand from a deck consisting of R ranks and W suits. A subset of J ranks can be the high rank in a straight or straight flush. If we allow Aces to be the low rank in a straight, J = R – 3, otherwise J = R – 4.

The total number of ways is $\binom{R\,W}{5}$

The various hand types are:

Royal flush	choose 1 rank from 1 (Ace) with 1 suit from W	$\binom{W}{1}$ ways
Ordinary straight flush	choose 1 rank from J – 1 with 1 suit from W	$\binom{J-1}{1}\binom{W}{1}$ ways
Four of a kind	choose 1 rank from R with 4 suits from W and 1 rank from R-1 with 1 suit from W	$\binom{R}{1}\binom{W}{4}\binom{R-1}{1}\binom{W}{1}$ ways
Full house	Full house, choose 1 rank from R with 3 suits from W and 1 rank from R-1 with 2 suits from W	$\binom{R}{1}\binom{W}{3}\binom{R-1}{1}\binom{W}{2}$ ways
Flush	Flush, choose 5 ranks from R with 1 suit from W but subtract the straight flushes	$[\binom{R}{5} - \binom{J}{1}]\binom{W}{1}$ ways
Straight	choose 1 rank from J with 1 suit from W (5 times) but subtract the straight flushes	$\binom{J}{1}[\binom{W}{1}^5 - \binom{W}{1}]$ ways
Three of a kind	Three of a kind, choose 1 rank from R with 3 suits from W and 2 ranks from R-1 with 1 suit from W (2 times)	$\binom{R}{1}\binom{W}{3}\binom{R-1}{2}\binom{W}{1}^2$ ways
Two pair	Two pair, choose 2 ranks from R with 2 suits from W (2 times) and 1 rank from R-2 with 1 suit from W	$\binom{R}{2}\binom{W}{2}^2\binom{R-2}{1}\binom{W}{1}$ ways
Pair	choose 1 rank from R with 2 suits from W and 3 ranks from R-1 with 1 suit from W (3 times)	$\binom{R}{1}\binom{W}{2}\binom{R-1}{3}\binom{W}{1}^3$ ways
No pair	choose 5 ranks from R with 1 suit from W (5 times) but subtract flushes, straights and straight flushes	$[\binom{R}{5} - \binom{J}{1}][\binom{W}{1}^5 - \binom{W}{1}]$ ways
Five of a kind	choose 1 rank from R with 5 suits from W	$\binom{R}{1}\binom{W}{5}$ ways

We can implement these formulas in a spreadsheet and are able to see the probabilities that the classic hand rankings are based on as well as investigate the effect of varying the number of ranks and suits. Notice that in Manila poker with a stripped deck (R = 8), flushes become rarer than full houses, which the game incorporates by adjusting the rankings accordingly.

	A	B	C	D	E	F	G	H	I	J	K	L
1				normal	2suits	3suits	5suits	6suits	manila	short	short	
2			R	13	13	13	13	13	8	4	3	
3			W	4	2	3	5	6	4	4	4	
5			J	10	10	10	10	10	4	0	0	
6												
7			gap?						1			
8												
9		total		2598960	65780	575757	8259888	21111090	201376	4368	792	
10		rf		4	2	3	5	6	4	0	0	
11		osf		36	18	27	45	54	12	0	0	
12		4x		624	0	0	3900	14040	224	48	24	
13		fh		3744	0	468	15600	46800	1344	288	144	
14		f		5108	2554	3831	6385	7662	208	0	0	
15		s		10200	300	2400	31200	77700	4080	0	0	
16		3x		54912	0	7722	214500	617760	10752	768	192	
17		2pair		123552	1716	23166	429000	1158300	24192	1728	432	
18		pair		1098240	22880	231660	3575000	9266400	107520	1536	0	
19		hc		1302540	38310	306480	3984240	9922290	53040	0	0	
20		5x		0	0	0	13	78	0	0	0	
21		SUM		2598960	65780	575757	8259888	21111090	201376	4368	792	
22		Discrepancy?		0	0	0	0	0	0	0	0	
23												
24												