MathsJam South

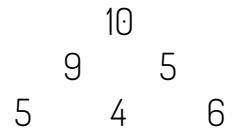
OMG22

October 9th, 2022

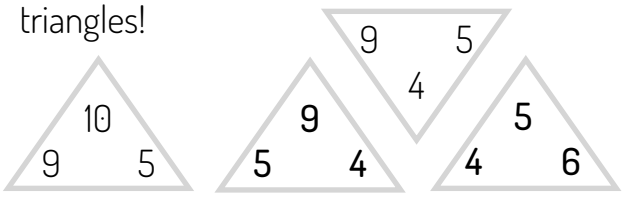
Fancy Problem

A fancy triangle is an equilateral triangular array of integers such that the sum of the three numbers in any unit equilateral triangle is a multiple of 3.

For example:



is a fancy triangle with three rows because the sum of the numbers in each of the following four unit equilateral triangles is a multiple of 3 - note we include upside down triangles!

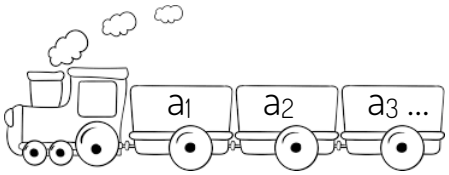


Suppose that a fancy triangle has ten rows. How many individual numbers in the triangle could be multiples of three?

Proof Problem

You might know the divisibility by 9 test:
a number is divisible by 9 if (and only if) the digits add to a multiple of 9.
Why not prove it?

Average Problem



A sequence $a_1, a_2, \ldots, a_{100}$ has integers in which the first and last terms are equal to 0.

Except for the first and last terms, each term a_i is larger than the average of its neighbours a_{i-1} and a_{i+1}.

What is the smallest possible value for the term a_{19}?

Alt Problem

Each term of an infinite sequence below is either 1 or 0.

$$a_1, a_2, \ldots, a_n, \ldots$$

No two consecutive terms add to the sum of the next two terms, and similarly for any three consecutive terms and the next three consecutive terms.

Prove that if $a_1 = 0$, then $a_{2020} = 1$

Hint Problem

For which problem is this a hint?

$$10 = 9 + 1,$$
$$100 = 99 + 1,$$
$$1000 = 999 + 1,$$
$$\ldots$$